EXERCISE 0.1

2/. (a) For $x = -1$ and $x = 4$, $y = 1$.

(b) There is no value of x for which $y = 3$.

(c) For $y = -1$, $x = 3$.

(d) For $x = 0, 3, 5$, $y \leq 0$.

(e) maximum value of $y = 9$ at $x = 6$
minimum value of $y = -2$ at $x = 0$.

3/. (a) It is a function because no vertical line intersects the graph more than once.

(b) It is a function because no vertical line intersects the graph more than once.

(c) It is not a function because the vertical line test fails.

(d) It is not a function because the vertical line test fails.

4/. (a) $f(x) = \dfrac{x^2+1}{x+1}$; $g(x) = x$

The natural domain of f consist of all real numbers x except $x = -1$ because at $x = -1$ the denominator becomes zero.

The [illegible] natural domain of $g(x)$ consists of all real numbers x.

Natural domain of $f = (-\infty, -1) \cup (-1, \infty)$

Natural domain of $g = (-\infty, \infty)$

Natural domain of f = Natural domain g if $x = -1$ is excluded from the natural of g. i.e. $f(x) = g(x)$ on $(-\infty, -1) \cup (-1, \infty)$.

(b) $f(x) = \dfrac{x\sqrt{x} + \sqrt{x}}{x+1}$; $g(x) = \sqrt{x}$

The function f have real values for all $x \geq 0$.

Natural domain of $f(x) = x \geq 0$ or $[0, +\infty)$

The function g have real values for all $x \geq 0$.

Natural domain of $g(x) = x \geq 0$ or $[0, +\infty)$.

Therefore Natural domain of f = Natural domain of g

7/ (a) $f(x) = 3x^2 - 2$

$f(0) = 3(0)^2 - 2 = -2$

$f(2) = 3(+2)^2 - 2 = 10$

$f(-2) = 3(-2)^2 - 2 = 10$

$f(3) = 3(3)^2 - 2 = 25$

$f(\sqrt{2}) = 3(\sqrt{2})^2 - 2 = 4$

$f(3t) = 3(3t)^2 - 2 = 27t^2 - 2$

(b)

(b) $f(x) = \begin{cases} \frac{1}{x}, & x > 3 \\ 2x & x \leq 3 \end{cases}$

$f(0) = 2(0) = 0$

$f(2) = 2(2) = 4$

$f(-2) = 2(-2) = -4$

$f(3) = 2(3) = 6$

$f(\sqrt{2}) = 2\sqrt{2}$

$$f(3t) = \begin{cases} \frac{1}{3t} & 3t > 3 \\ 2(3t) & 3t \leq 3 \end{cases}$$

$$f(3t) = \begin{cases} \frac{1}{3t} & t > 1 \\ 6t & t \leq 1 \end{cases}$$

8/

(a) $g(x) = \frac{x+1}{x-1}$

$$g(t^2-1) = \frac{(t^2-1)+1}{(t^2-1)-1} = \frac{t^2-1+1}{t^2-1-1} = \frac{t^2}{t^2-2}$$

(b) $$g(x) = \begin{cases} \sqrt{x+1} & x \geq 1 \\ 3 & x < 1 \end{cases}$$

$$g(t^2-1) = \begin{cases} \sqrt{t^2-1+1} & t^2-1 \geq 1 \\ 3 & t^2-1 < 1 \end{cases}$$

$$g(t^2-1) = \begin{cases} \sqrt{t^2} & t^2 \geqslant 1+1 \\ 3 & t^2 < 1+1 \end{cases}$$

$$g(t^2-1) = \begin{cases} |t|, & t^2 \geqslant 2 \\ 3 & t^2 < 2 \end{cases}$$

9/ (a) $f(x) = \dfrac{1}{x-3}$

The function have real values for all x exept $x = +3$.

Natural domain of f : $x \neq 3$ or $(-\infty, 3) \cup (3, +\infty)$

Range of f = $y \geqslant 0$.

(b) $F(x) = \dfrac{x}{|x|}$

The natural domain of $F(x)$ consists of all real numbers x except $x = 0$.

Rang : $\{-1, 1\}$

For negative values of x ~~F~~ $F = -1$ and for positive values of x, $F = 1$.

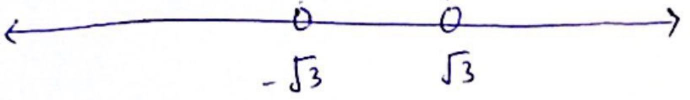
(C) $g(x) = \sqrt{x^2-3}$

$g(x) = \sqrt{(x-\sqrt{3})(x+\sqrt{3})}$

The natural domain of g consists of all real numbers x for which $(x-\sqrt{3})(x+\sqrt{3}) \geqslant 0$.

$x-\sqrt{3}$: - - - - - - - - - - - + + + + + + +

$x+\sqrt{3}$: - - - - - - + + + + + + + + + + + +



EXERCISE 0.1

Natural domain : $(-\infty, -\sqrt{3}] \cup [\sqrt{3}, +\infty)$.

Range : $y \geq 0$.

(d) $G(x) = \sqrt{x^2-2x+5}$

$x^2-2x+5 = x^2-2x+1+4$

$= (x-1)^2+4 \geq 4$

$G(x) = \sqrt{x^2-2x+5} = \sqrt{(x-1)^2+4} \geq \sqrt{4} = 2$

$G(x) \geq 2$. for all x.

$\therefore$ $G(x)$ is defined for all x.

Natural domain : all Real numbers x

Range : $y \geq 2$

(e) $h(x) = \frac{1}{1-\sin x}$

$h(x)$ is defined for x for which $1-\sin x \neq 0$

$\sin x \neq 1 \Rightarrow x \neq \sin^{-1}(1)$

$\Rightarrow$ $x \neq$ all ~~odd~~ odd multiples of $\frac{\pi}{2}$.

$\Rightarrow$ $x \neq (2n+\frac{1}{2})\frac{\pi}{2}$, $n = 0, \pm 1, \pm 2, \ldots$

and $-1 \leq \sin x < 1$ for such values of x.

$\Rightarrow$ ~~$-1 \neq 1 \leq$~~ $-1 < -\sin x \leq 1$ multiplied by -1.

$-1+1 < 1-\sin x \leq 1+1$

$\Rightarrow$ $0 < 1-\sin x \leq 2$

$$\Rightarrow \quad 1-\sin x \le 2$$

$$\Rightarrow \quad \frac{1}{1-\sin x} \geq \frac{1}{2}$$

$$\Rightarrow \quad h(x) \geq \frac{1}{2}$$

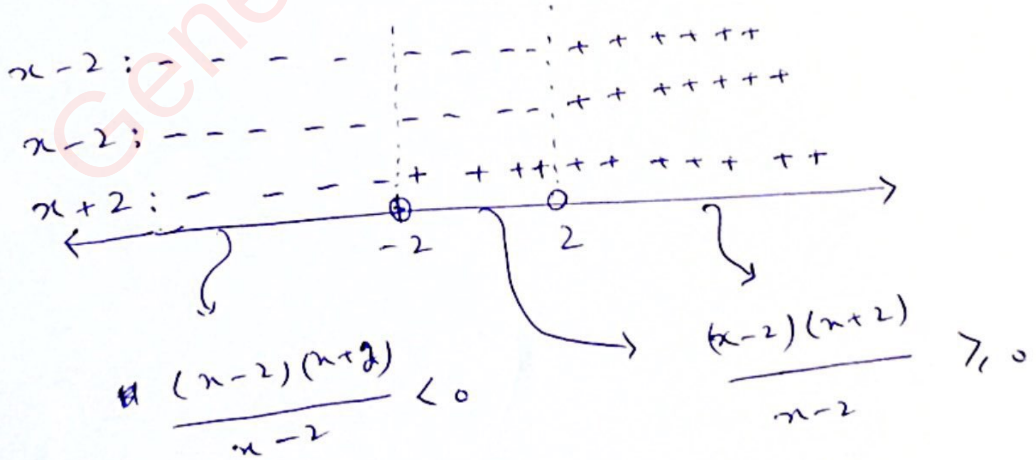
$\therefore$ Range : $y \geq \frac{1}{2}$.

(f) $H(x) = \sqrt{\dfrac{x^2-4}{x-2}}$

$$H(x) = \sqrt{\frac{(x-2)(x+2)}{x-2}}$$

$H(x)$ is undefined at $x=2$ because the denominator becomes zero at $x=2$.

The natural domain of H is consists of all real numbers x for which $\dfrac{(x-2)(x+2)}{x-2} \geq 0$



Natural domain of H : $[-2, 2) \cup (2, +\infty)$

Range : $y \geq 0$ or $[0, +\infty)$.

EXERCISE 0.1

10/ (a) $f(x) = \sqrt{3-x}$

The natural domain of consists of all x for which

$f(x) \geq 0 \Rightarrow \quad 3 - x \geq 0$

$\Rightarrow 3 \geq x \quad$ or $\quad x \leq 3$

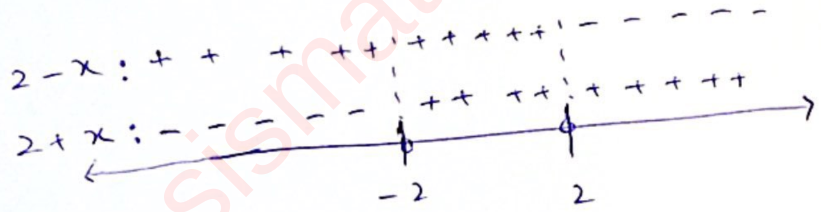
Range : $y \geq 0$ or $[0, +\infty)$

(b) $F(x) = \sqrt{4 - x^2}$

$F(x) = \sqrt{(2-x)(2+x)}$

$2-x$ becomes zero at $x = 2$ and

$2+x$ becomes zero at $x = -2$.



Natural domain of F consists of all real numbers x for which $(2-x)(2+x) \geq 0$.

Natural domain : $-2 \leq x \leq 2$

Range : $[0, 2]$ or $0 \leq y \leq 2$.

(c) $g(x) = 3 + \sqrt{x}$

Natural domain of g consists of all real numbers x for which $x \geq 0$.

Natural domain : $[0, \infty)$ or $x \geq 0$

Range : $y \geq 3$

(d) $G(x) = x^3 + 2$

G have real values for all x

Natural domain : all x or $(-\infty, +\infty)$

Range : all y or $(-\infty, +\infty)$.

(e) $h(x) = 3 \sin x$

$h(x)$ have real values of all x.

Natural domain : all x

Range : As $-1 \leq \sin x \leq 1$

Multiplying by 3

$-3 \leq 3 \sin x \leq 3$

Range : $-3 \leq y \leq 3$.

(f) $H(x) = (\sin \sqrt{x})^{-2}$

$\sqrt{x}$ is defined only for $x \geq 0$.

$H(x)$ ~~have~~ is defined for all x such that $\sin \sqrt{x} \neq 0$

$\Rightarrow \sqrt{x} = \sin^{-1}(0)$

$\Rightarrow \sqrt{x} = n\pi$ for $n =$ ~~0~~, 1, 2, 3 ...

$x \neq (n\pi)^2$ for $n =$ ~~0~~ 1, 2, ...

Natural domain : $x > 0$, $x \neq (n\pi)^2$ for $n = 1, 2, 3 \ldots$

For these values of x, we have

$$0 < |\sin\sqrt{x}| \leq 1$$

$$\Rightarrow \quad 0 < (\sin\sqrt{x})^2 \leq 1$$

$$\Rightarrow \quad (\sin\sqrt{x})^2 \leq 1$$

$$\Rightarrow \quad \frac{1}{(\sin\sqrt{x})^2} \geq 1$$

$$\Rightarrow \quad H(x) \geq 1$$

Range : $y \geq 1$.

15/ It is a function. The graph in a upper portion of the circle $x^2+y^2 = 25$

$$y(x) = \sqrt{25-x^2}$$

16/ yes, it is a function. The graph is a lower portion of the circle $x^2+y^2=25$

$$y(x) = -\sqrt{25-x^2}$$

17/ yes it is a function

$$y(x) = \begin{cases} \sqrt{25-x^2}, & -5 \leq x \leq 0 \\ -\sqrt{25-x^2} & 0 < x \leq 5 \end{cases}$$

EXERCISE 0·1

18/ It is not a graph because the vertical line test fails.

23/ (a) $y = x^2 - 6x + 8$

For $y=0$, $x^2 - 6x + 8 = 0$

$$x^2 - 4x - 2x + 8 = 0$$

$$x(x-4) - 2(x-4) = 0$$

$$(x-4)(x-2) = 0$$

$$x - 4 = 0 \qquad x - 2 = 0$$

$$x = 4 \qquad x = 2$$

(b) $y = -10$

$$x^2 - 6x + 8 = -10$$

$$x^2 - 6x + 8 + 10 = 0$$

$$x^2 - 6x + 18 = 0$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(18)}}{2(1)}$$

$$x = \frac{+6 \pm \sqrt{36 - 72}}{2}$$

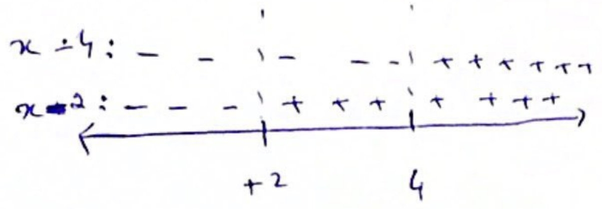
$$x = \frac{6 \pm \sqrt{-36}}{2}$$ which is not real number

The answer is none.

(c) For $y \geq 0$

$x^2 - 6x + 8 \geq 0$

$(x-4)(x-2) \geq 0$



$x \leq 2$; $x \geq 4$.

(d)

$y = x^2 - 6x + 8$

$y = x^2 - 6x + 8 + 1 - 1$

$y = x^2 - 6x + 9 - 1$

$y = (x-3)^2 - 1$

minimum value of $y = -1$, no maximum value

24, $y = 1 + \sqrt{x}$

(a) For $y = 4$

$1 + \sqrt{x} = 4$

$\sqrt{x} = 4 - 1$

$\sqrt{x} = 3$

$x = 9$

(b) For $y = 0$

$1 + \sqrt{x} = 0$

$\sqrt{x} = -1$, therefore answer is none.

(c) For $y \geq 6$

$1+\sqrt{x} \geq 6$

$\sqrt{x} \geq 5$

$x \geq 25$

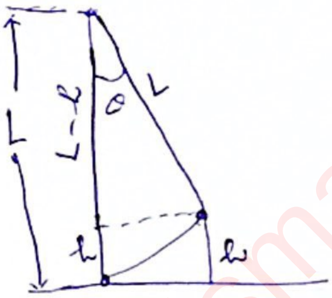
(d) $y = \sqrt{x} + 1$

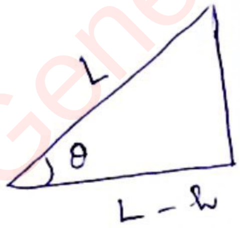
$\sqrt{x}$ exists only for $x \geq 0$

∴ minimum value of $y = 1$

no maximum value.

25)





$$\cos\theta = \frac{L-h}{L}$$

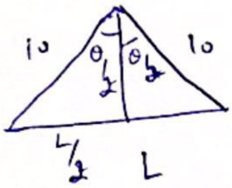
$L\cos\theta = L - h$

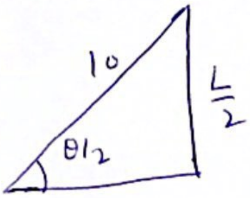
$h = L - L\cos\theta$

$h = L(1 - \cos\theta)$

$h = L(1 - \cos\theta)$

26/.





$$\sin \theta/2 = \frac{L/2}{10}$$

$$\frac{L}{2} = 10 \sin \theta/2$$

$$L = 20 \sin \theta/2$$

27/.(a) $f(x) = |x| + 3x+1$

As $|x| = \begin{cases} -x, & x < 0 \\ x & x \geq 0 \end{cases}$

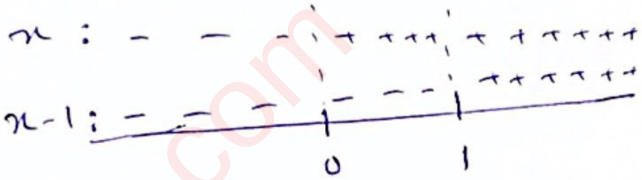
$$\therefore \quad f(x) = \begin{cases} -x+3x+1, & x < 0 \\ x+3x+1, & x \geq 0 \end{cases}$$

$$f(x) = \begin{cases} 2x+1, & x < 0 \\ 4x+1 & x \geq 0 \end{cases}$$

27 (b). $g(x) = |x| + |x-1|$

$$|x| = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$|x-1| = \begin{cases} -(x-1), & x-1<0 \text{ or } x<1 \\ x-1, & x-1 \geq 0 \text{ or } x \geq 1 \end{cases}$$



$$\therefore \quad g(x) = \begin{cases} -x-(x-1), & x<0 \\ -(x-1)+x, & 0 \leq x < 1 \\ x+(x-1), & x \geq 1 \end{cases}$$

$$g(x) = \begin{cases} -2x+1, & x<0 \\ 1, & 0 \leq x<1 \\ 2x-1, & x \geq 1. \end{cases}$$

28/ (a) $f(x) = 3 + |2x-5|$

$$|2x-5| = \begin{cases} -(2x-5), & 2x-5<0 \text{ or } x < 5/2 \\ 2x-5, & 2x-5 \geq 0 \text{ or } x \geq 5/2 \end{cases}$$

$$f(x) = \begin{cases} 3-(2x-5), & x < 5/2 \\ 3+(2x-5), & x \geq 5/2 \end{cases}$$

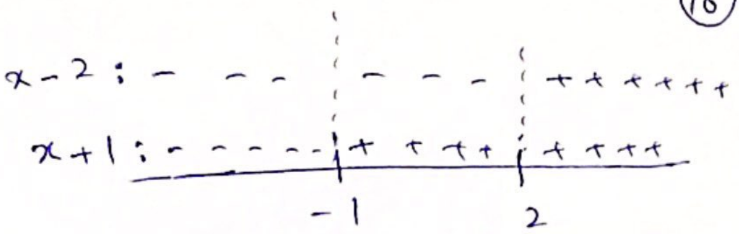
$$f(x) = \begin{cases} -2x+8, & x < 5/2 \\ 2x-2, & x \geq 5/2 \end{cases}$$

(b) $g(x) = 3|x-2| - |x+1|$

$$|x-2| = \begin{cases} -(x-2), & x-2<0 \text{ or } x<2 \\ x-2, & x-2 \geq 0 \text{ or } x \geq 2 \end{cases}$$

$$|x+1| = \begin{cases} -(x+1), & x+1<0 \text{ or } x<-1 \\ x+1, & x+1 \geq 0 \text{ or } x \geq -1 \end{cases}$$

EXERCISE 0.1



$$g(x) = -3(x-2) - (-(x+1)), \quad x < -1$$

$$= -3x + 6 + x + 1, \quad x < -1$$

$$g(x) = -2x + 7, \quad x < -1$$

$$g(x) = -3(x-2) - (x+1), \quad -1 \leq x < 2$$

$$g(x) = -3x + 6 - x - 1, \quad -1 \leq x < 2$$

$$g(x) = -4x + 5, \quad -1 \leq x < 2$$

$$g(x) = 3(x-2) - (x+1), \quad x \geq 2$$

$$g(x) = 3x - 6 - x - 1, \quad x \geq 2$$

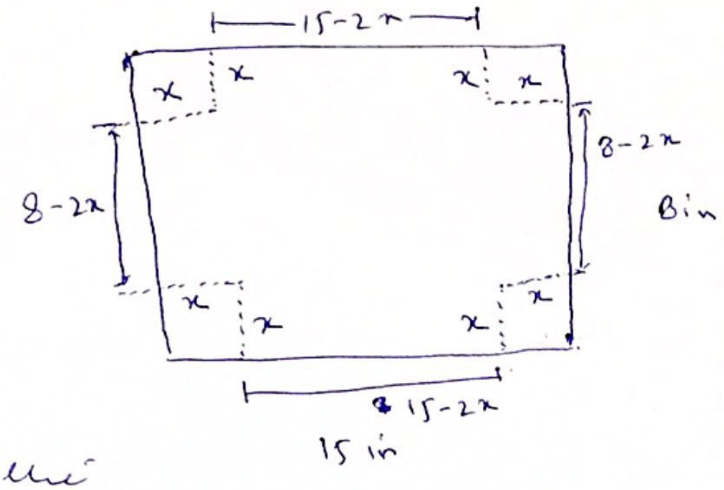
$$g(x) = 2x - 7, \quad x \geq 2$$

Hence

$$g(x) = \begin{cases} -2x + 7, & x < -1 \\ -4x + 5, & -1 \leq x < 2 \\ 2x - 7, & x \geq 2 \end{cases}$$

EXERCISE 0.1

29/



Dimension of the box are 8 - 2x by $\overset{15}{\cancel{30}} - 2x$ by x (as shown in the figure)

V ~~= 30-2x~~

$V = (15-2x)(8-2x)(x)$

$V = (120 - 30x - 16x + 4x^2)x$

$V = 120x - 46x^2 + 4x^3$

(b) We cannot cut out squares of sides length x more than 4. therefore

Domain : $0 < x < 4$

30/ (a) length = $6-2x$

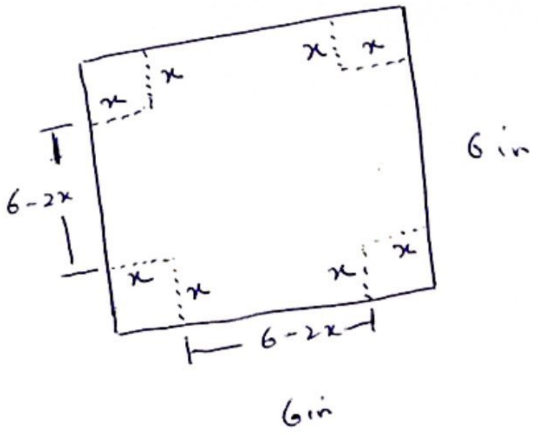
width = $6-2x$

height = x

V = length × width × height

$V = (6-2x)(6-2x)(x)$

$V = (6-2x)^2 x$

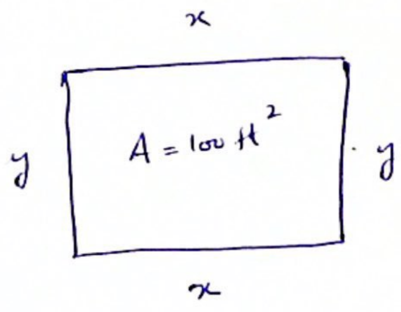


$$V = (36 - 24x + 4x^2)x$$

$$V = 36x - 24x^2 + 4x^3$$

(b) As each side is of 6 in, therefore we can not cut out squares of sides of length x more than 3.

$\therefore$ Domain : $0 < x < 3$.



31/ Let x be the

(a) length of the side adjacent to the building.

$$L = x + 2y \quad — ①$$

(b) $A = xy = 1000$

$$xy = 1000$$

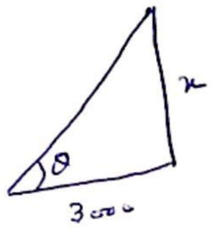
$$y = \frac{1000}{x}$$

Substituting $y = \frac{1000}{x}$ into ①

$$L = x + \frac{2000}{x}$$

(c) $0 < x < 100$[illegible]

32/ From the figure



(a) $\tan\theta = \frac{x}{3000}$

$$x = 3000\tan\theta$$

32 (b) $0 \leq \theta < \pi/2$

(c) $\theta = \pi/4$

$x = 3000 \tan \pi/4$

$x = 3000(1) = 3000$ ft.

35/ $f(x) = \frac{(x+2)(x^2-1)}{(x+2)(x-1)}$

(i) $x = -2, 1$ are values at which holes occur, because the denominator becomes zero at these values

(ii) $f(x) = \frac{(x+2)(x-1)(x+1)}{(x+2)(x-1)}$

$f(x) = x+1 = g(x)$ say

$g(x) = x+1.$

36/ $f(x) = \frac{x^2+|x|}{|x|}$

(i) $x = 0$ is the value at which hole occurs. because denominator becomes zero at this value.

(ii) $f(x) = \frac{|x|^2 + |x|}{|x|} = \frac{|x|(|x|+1)}{|x|} = |x|+1$

so $g(x) = |x|+1$