1/ (a) $t = 10s$, Instantaneous velocity = ?

From the graph we see that

at $t_1 = 5s$, $S_1 = 10m$

at $t_2 = 15s$ $S_2 = 50m$

$$m_{tan} = \frac{S_2 - S_1}{t_2 - t_1} = \frac{50 - 10}{15 - 5} = \frac{40}{10} = 4m/s$$

2/ Instantaneous velocity = ? at $t = 4s$ and $t = 8s$.

From the graph we see that

at $t_1 = 2s$, $S_1 = 0$

at $t_2 = 10s$, $S_2 \approx 90m$

at $t = 4s$,

$$m_{tan} = \frac{S_2 - S_1}{t_2 - t_1}$$

$$m_{tan} = \frac{90 - 0}{10 - 2} = \frac{90}{8} = \frac{45}{4} m/s.$$

From the graph we have

at $t_1 = 4s$, $S_1 = 0$

at $t_2 = 10s$, $S_2 \approx 140m$

at $t = 8s$, $m_{tan} = \frac{S_2 - S_1}{t_2 - t_1} \approx \frac{140 - 0}{10 - 4} = \frac{140}{6} = \frac{70}{3} m/s$

$$m_{tan} \approx \frac{70}{3} m/s$$

3/ The accompanying figure shows the position versus time curve.... -

(a) the average velocity over the interval $0 \leq t \leq 3$

From the graph, we have

at $t_1 = 0$, $S_1 = 10$ cm

at $t_2 = 3$ s, $S_2 = 10$ cm

$$V_{ave} = \frac{S_2 - S_1}{t_2 - t_1} = \frac{10-10}{3-0} = 0 \text{ cm/s}$$

(b) the values of t at which the instantaneous velocity is zero.

The points where the tangent line is horizontal i.e the points at which slope of the tangent line is zero.

i.e Slope of tangent line = Instantaneous velocity

From the graph we see that at $t = 0$, $t = 2$, $t = 4$. and $t = 8$, the instantaneous velocity zero (at these points tangent lines are horizontal).

(c) the values of t at which the instantaneous velocity is either maximum or minimum.

From the graph we have

at $t = 1$, the slope is +ve and the instantaneous velocity is maximum.

at $t = 3$, the slope is −ve and the instantaneous velocity is minimum.

(d) the instantaneous velocity when $t = 3s$

at $t_1 = 2s$, $S_1 = 15\,cm$

$t_2 = 4s$, $S_2 = 5\,cm$

$$\text{Instantaneous velocity} \approx \frac{S_2 - S_1}{t_2 - t_1} = \frac{5 - 15}{4 - 2} = \frac{-10}{2} = -5\,cm/s$$

4/ The accompanying figure shows the position versus time curves ...

(a) Since the slope of the tangent line is decreasing with increasing times therefore the instantaneous velocity is decreasing

(b) Since the slope of the tangent line increases with time, therefore the instantaneous velocity is increasing

(c) Since the slope of the tangent line increases with time therefore the instantaneous velocity is increasing.

(d) Since the slope of the tangent line is decreasing with increasing time, therefore the instantaneous velocity is decreasing.

EXERCISE 2.1

11-14 A function $y = f(x)$ and the values of x_0 and x_1 are given.

(a) Find the average rate of change ---

11/ $y = 2x^2$; $x_0 = 0$, $x_1 = 1$

$y = f(x) = 2x^2$

(a) average rate of change

$$r_{ave} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$r_{ave} = \frac{f(1) - f(0)}{1 - 0} = \frac{2(1)^2 - 0}{1 - 0} = 2$$

(b) Instantaneous rate of change (m_{tan})

$$m_{tan} = r_{inst} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to 0} \frac{2x_1^2 - f(0)}{x_1 - 0} \qquad \because x_0 = 0,\ f(x_1) = 2x_1^2$$

$$= \lim_{x_1 \to 0} \frac{2x_1^2}{x_1} = \lim_{x_1 \to 0} 2x_1 = 2(0) = 0$$

(c)

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{2x_1^2 - 2x_0^2}{x_1 - x_0} \qquad \because f(x_1) = 2x_1^2,\ f(x_0) = 2x_0^2$$

$$= \lim_{x_1 \to x_0} \frac{2(x_1^2 - x_0^2)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{2(x_1 + x_0)(x_1 - x_0)}{x_1 - x_0}$$

$$m_{tan} = \lim_{x_1 \to x_0} 2(x_1 + x_0)$$

$$= 2(x_0 + x_0) = 4x_0$$

12/ $y = x^3$; $x_0 = 1$, $x_1 = 2$

$y = f(x) = x^3$

(a) average rate of change

$$r_{ave} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$r_{ave} = \frac{f(2) - f(1)}{2 - 1} \qquad \therefore x_1 = 2,\ x_0 = 1$$

$$r_{ave} = \frac{(2)^3 - (1)^3}{1} = 8 - 1 = 7 \qquad \therefore f(2) = 2^3,\ f(1) = 1^3$$

(b) Instantaneous rate of change

$$m_{tan} = r_{inst} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to 1} \frac{f(x_1) - f(1)}{x_1 - 1} \qquad \therefore x_0 = 1,\ f(x_1) = x_1^3,\ f(1) = 1$$

$$= \lim_{x_1 \to 1} \frac{x_1^3 - 1}{x_1 - 1} \qquad \therefore a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= \lim_{x_1 \to 1} \frac{(x_1 - 1)(x_1^2 + x_1 + 1)}{x_1 - x_1}$$

$$= \lim_{x_1 \to 1} (x_1^2 + x_1 + 1) = 1^2 + 1 + 1 = 3$$

(c) $$m_{\tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

As $f(x) = x^3$

$\therefore$ $f(x_1) = x_1^3$

$f(x_0) = x_0^3$

$$= \lim_{x_1 \to x_0} \frac{x_1^3 - x_0^3}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{(x_1 - x_0)(x_1^2 + x_0 x_1 + x_0^2)}{x_1 - x_0}$$

$a^3 - b^3$
$= (a-b)(a^2 + ab + b^2)$

$$= \lim_{x_1 \to x_0} (x_1^2 + x_0 x_1 + x_0^2)$$

$$= x_0^2 + x_0 x_0 + x_0^2 = 3x_0^2$$

13/ $y = \frac{1}{x}$; $x_0 = 2$, $x_1 = 3$

$y = f(x) = 1/x$

(a) average rate of change

$$r_{ave} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$\therefore$ $x_1 = 3$, $x_0 = 2$

$$r_{ave} = \frac{f(3) - f(2)}{3 - 2}$$

$\therefore$ $f(x) = 1/x$

$f(3) = 1/3$

$f(2) = 1/2$

$$= \frac{1/3 - 1/2}{1}$$

$$= \frac{1}{3} - \frac{1}{2} = \frac{2-3}{6} = -\frac{1}{6}$$

(b) Instantaneous rate of change

$$m_{\tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$x_0 = 2$

$\therefore f(x) = 1/x$

$f(x_1) = 1/x_1$

$f(2) = 1/2$

$$= \lim_{x_1 \to 2} \frac{f(x_1) - f(2)}{x_1 - 2}$$

$$= \lim_{x_1 \to 2} \frac{1/x_1 - \frac{1}{2}}{x_1 - 2}$$

$$= \lim_{x_1 \to 2} \frac{\frac{2 - x_1}{2x_1}}{x_1 - 2}$$

$$= \lim_{x_1 \to 2} \frac{2 - x_1}{2x_1(x_1 - 2)} = \lim_{x_1 \to 2} \frac{-(x_1 - 2)}{2x_1(x_1 - 2)}$$

$$= \lim_{x_1 \to 2} -\frac{1}{2x_1} = -\frac{1}{2(2)} = -1/4$$

(c)

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$\therefore f(x) = 1/x$

$f(x_1) = 1/x_1$

$f(x_0) = 1/x_0$

$$= \lim_{x_1 \to x_0} \frac{1/x_1 - 1/x_0}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{\frac{x_0 - x_1}{x_1 x_0}}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{x_0 - x_1}{x_1 x_0 (x_1 - x_0)}$$

$$= \lim_{x_1 \to x_0} \frac{-(x_1 - x_0)}{x_1 x_0 (x_1 - x_0)}$$

$$= \lim_{x_1 \to x_0} -\frac{1}{x_1 x_0} = -\frac{1}{x_0 x_0} = -1/x_0^2$$

14/ $y = 1/x^2$; $x_0 = 1$, $x_1 = 2$

$y = f(x) = 1/x^2$

$f(x_0) = 1/x_0^2$, $f(x_1) = 1/x_1^2$

$f(1) = 1/1^2 = 1$, $f(2) = 1/2^2 = 1/4$

(a) average rate of change (m_{sec}

$$m_{sec} = r_{ave} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \frac{f(2) - f(1)}{2 - 1}$$

$$= \frac{1/2^2 - 1/1^2}{1} = 1/4 - 1 = \frac{1-4}{4} = -3/4$$

(b) Instantaneous rate of change

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to 1} \frac{f(x_1) - f(1)}{x_1 - 1} \qquad \because x_0 = 1$$

$$= \lim_{x_1 \to 1} \frac{1/x_1^2 - 1}{x_1 - 1} \qquad \because f(1) = 1$$

$$= \lim_{x_1 \to 1} \frac{\frac{1 - x_1^2}{x_1^2}}{x_1 - 1}$$

$$= \lim_{x_1 \to 1} \frac{-(x_1^2 - 1)}{x_1^2 (x_1 - 1)}$$

$$m_{tan} = \lim_{x_1 \to 1} \frac{-(x_1-1)(x_1+1)}{x_1^2(x_1-1)} \qquad \therefore x_1^2 - 1 = (x_1+1)(x_1-1)$$

$$m_{tan} = \lim_{x_1 \to 1} -\frac{(x_1+1)}{x_1^2} = -\frac{(1+1)}{1^2} = -2$$

(C)

$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{1/x_1^2 - 1/x_0^2}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{\frac{x_0^2 - x_1^2}{x_1^2 x_0^2}}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{x_0^2 - x_1^2}{x_1^2 x_0^2 (x_1 - x_0)}$$

$$= \lim_{x_1 \to x_0} \frac{(x_0 - x_1)(x_0 + x_1)}{x_1^2 x_0^2 (x_1 - x_0)}$$

$$= \lim_{x_1 \to x_0} \frac{-(x_1 - x_0)(x_0 + x_1)}{x_1^2 x_0^2 (x_1 - x_0)}$$

$$= \lim_{x_1 \to x_0} \frac{-(x_0 + x_1)}{x_1^2 x_0^2}$$

$$= -\frac{(x_0 + x_0)}{x_0^2 x_0^2} = -\frac{2x_0}{x_0^4} = -\frac{2}{x_0^3}$$

15–18. A function $y = f(x)$ and an x-value x_0 are given.

(a) Find a formula for the slope of the tangent line to the graph of f at a general point $x = x_0$

(b) Use the formula obtained in part (a) to find the slope of the tangent line for the given value of x_0.

15/ $f(x) = x^2 - 1;\ x_0 = -1$

(a)
$$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$\because f(x_1) = x_1^2 - 1$
$f(x_0) = x_0^2 - 1$

$$= \lim_{x_1 \to x_0} \frac{(x_1^2 - 1) - (x_0^2 - 1)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{x_1^2 - 1 - x_0^2 + 1}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{x_1^2 - x_0^2}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{(x_1 + x_0)(x_1 - x_0)}{x_1 - x_0}$$

$$m_{tan} = \lim_{x_1 \to x_0} (x_1 + x_0) = x_0 + x_0 = 2x_0$$

(b)
$$m_{tan} = 2x_0$$

Put $x_0 = -1$

$\therefore m_{tan} = 2(-1) = -2$

16/ $f(x) = x^2 + 3x; \quad x_0 = 2$

(a) $m_{tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$\therefore f(x_1) = x_1^2 + 3x_1$

$f(x_0) = x_0^2 + 3x_0$

$m_{tan} = \lim_{x_1 \to x_0} \dfrac{(x_1^2 + 3x_1) - (x_0^2 + 3x_0)}{x_1 - x_0}$

$m_{tan} = \lim_{x_1 \to x_0} \dfrac{x_1^2 + 3x_1 - x_0^2 - 3x_0}{x_1 - x_0}$

$m_{tan} = \lim_{x_1 \to x_0} \dfrac{x_1^2 - x_0^2 + 3(x_1 - x_0)}{x_1 - x_0}$

$m_{tan} = \lim_{x_1 \to x_0} \dfrac{(x_1 + x_0)(x_1 - x_0) + 3(x_1 - x_0)}{x_1 - x_0}$

$m_{tan} = \lim_{x_1 \to x_0} \dfrac{(x_1 - x_0)(x_1 + x_0 + 3)}{x_1 - x_0}$

$m_{tan} = \lim_{x_1 \to x_0} (x_1 + x_0 + 3)$

$m_{tan} = x_0 + x_0 + 3 = 2x_0 + 3$

(b) $m_{tan} = 2x_0 + 3; \quad x_0 = 2$

$m_{tan} = 2(2) + 3 = 7$

17) $f(x) = x + \sqrt{x}$; $x_0 = 1$

(a) $m_{tan} = \lim_{x_1 \to x_0} \dfrac{f(x_1) - f(x_0)}{x_1 - x_0}$

$m_{tan} = \lim_{x_1 \to x_0} \dfrac{x_1 + \sqrt{x_1} - (x_0 + \sqrt{x_0})}{x_1 - x_0}$ $\quad \because f(x_1) = x_1 + \sqrt{x_1}$, $f(x_0) = x_0 + \sqrt{x_0}$

$m_{tan} = \lim_{x_1 \to x_0} \dfrac{x_1 + \sqrt{x_1} - x_0 - \sqrt{x_0}}{x_1 - x_0}$

$m_{tan} = \lim_{x_1 \to x_0} \dfrac{x_1 - x_0 + \sqrt{x_1} - \sqrt{x_0}}{x_1 - x_0}$

$= \lim_{x_1 \to x_0} \dfrac{x_1 - x_0}{x_1 - x_0} + \dfrac{\sqrt{x_1} - \sqrt{x_0}}{x_1 - x_0}$

$= \lim_{x_1 \to x_0} 1 + \dfrac{\sqrt{x_1} - \sqrt{x_0}}{x_1 - x_0}$

$m_{tan} = \lim_{x_1 \to x_0} \left(1 + \dfrac{\sqrt{x_1} - \sqrt{x_0}}{(\sqrt{x_1} + \sqrt{x_0})(\sqrt{x_1} - \sqrt{x_0})} \right)$ $\quad \because a^2 - b^2 = (a+b)(a-b)$

$\therefore x_1 - x_0 = (\sqrt{x_1} + \sqrt{x_0})(\sqrt{x_1} - \sqrt{x_0})$

$m_{tan} = \lim_{x_1 \to x_0} \left(1 + \dfrac{1}{\sqrt{x_1} + \sqrt{x_0}} \right)$

$m_{tan} = 1 + \dfrac{1}{\sqrt{x_0} + \sqrt{x_0}} = 1 + \dfrac{1}{2\sqrt{x_0}}$

(b) $m_{tan} = 1 + \dfrac{1}{2\sqrt{x_0}}$; $x_0 = 1$

$m_{tan} = 1 + \dfrac{1}{2\sqrt{1}} = 1 + \dfrac{1}{2} = 3/2$

EXERCISE 2.1

18/ $f(x) = \frac{1}{\sqrt{x}}$; $x_0 = 4$

(a) $$m_{tan} = \lim_{x_1 \to x_0} \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$= \lim_{x_1 \to x_0} \frac{1/\sqrt{x_1} - 1/\sqrt{x_0}}{x_1 - x_0}$$ $\because f(x_1) = 1/\sqrt{x_1}$, $f(x_0) = 1/\sqrt{x_0}$

$$m_{tan} = \lim_{x_1 \to x_0} \frac{\frac{\sqrt{x_0} - \sqrt{x_1}}{\sqrt{x_0}\sqrt{x_1}}}{x_1 - x_0}$$

$$m_{tan} = \lim_{x_1 \to x_0} \frac{-(\sqrt{x_1} - \sqrt{x_0})}{\sqrt{x_0 x_1}\,(x_1 - x_0)}$$

$$m_{tan} = \lim_{x_1 \to x_0} \frac{-(\sqrt{x_1} - \sqrt{x_0})}{\sqrt{x_0 x_1}\,(\sqrt{x_1} + \sqrt{x_0})(\sqrt{x_1} - \sqrt{x_0})}$$

$\because a^2 - b^2 = (a+b)(a-b)$

$(x_1 - x_0) = (\sqrt{x_1} + \sqrt{x_0})(\sqrt{x_1} - \sqrt{x_0})$

$$m_{tan} = \lim_{x_1 \to x_0} \frac{-1}{\sqrt{x_0 x_1}\,(\sqrt{x_1} + \sqrt{x_0})}$$

$$= \frac{-1}{\sqrt{x_0 x_0}\,(\sqrt{x_0} + \sqrt{x_0})} = \frac{-1}{\sqrt{x_0^2}\,(2\sqrt{x_0})}$$

$$m_{tan} = \frac{-1}{2\sqrt{x_0^3}} = \frac{-1}{2x_0^{3/2}}$$

(b) $$m_{tan} = -\frac{1}{2\sqrt{x_0^3}}, \quad x_0 = 4$$

$$m_{tan} = \frac{-1}{2(4^3)^{1/2}} = \frac{-1}{2(2^2)^{3/2}} = \frac{-1}{2(2)^3} = \frac{-1}{16}$$

26/ An object is released from rest (its initial velocity is zero) from the Empire state building at a height of 1250 ft above street level. The height of the object is modeled by the position function $S = f(t) = 1250 - 16t^2$

(a) verify that the object is still falling at $t = 5s$.

find t when an object hits the ground then

$S = 0 \Rightarrow 1250 - 16t^2 = 0$

$16t^2 = 1250 \Rightarrow t^2 = \frac{1250}{16}$

$t^2 = 78.125$

$t = \sqrt{78.125} = 8.8388 > 5$

$\Rightarrow$ at $t = 5s$ the object was falling.

(b) verify the average velocity of the object over the time interval from $t = 5$ to $t = 6s$.

$V_{ave} = \frac{f(6) - f(5)}{6-5}$

$V_{ave} = \frac{674 - 850}{1} = -176$ ft/s

$f(t) = 1250 - 16t^2$

$f(6) = 1250 - 16(6)^2 = 674$

$f(5) = 1250 - 16(5)^2 = 850$

(C) Find the object's instantaneous velocity at time $t = 5s$

$V_{inst} = \lim_{t_1 \to 5} \frac{f(t_1) - f(5)}{t_1 - 5}$

$\because f(t) = 1250 - 16t^2$

$f(t_1) = 1250 - 16t_1^2$

$= \lim_{t_1 \to 5} \frac{1250 - 16t_1^2 - 850}{t_1 - 5}$

$$V_{inst} = \lim_{t_1 \to 5} \frac{-16t_1^2 + 400}{t_1 - 5}$$

$$= \lim_{t_1 \to 5} \frac{-16(t_1^2 - 25)}{t_1 - 5}$$

$$= \lim_{t_1 \to 5} \frac{-16(t_1+5)(t_1-5)}{(t_1-5)} \quad \therefore \; t_1^2 - 25 = (t_1+5)(t_1-5)$$

$$= \lim_{t_1 \to 5} -16(t_1+5) = -16(5+5) = -160 \text{ ft/s}$$

27/ During the first 40s of a rocket flight, the rocket is propelled straight up so that in t seconds it reaches a height of $s = 0.3t^3$ ft

(a) How high does the rocket travel in 40s?

$$S = 0.3t^3; \quad t = 40s$$

$$S = 0.3(40)^3 = 0.3(64000) = 19200 \text{ ft.}$$

(b) What is the average velocity of the rocket during the first 40s?

$$S = 0.3t^3, \quad t_1 = 0, \quad t_2 = 40 \text{ s}$$

$$V_{ave} = \frac{S(t_2) - S(t_1)}{t_2 - t_1} =$$

$S(t_1) = 0$

$S(t_2) = 0.3\,t_2^3$

$$V_{ave} = \frac{0.3\,t_2^3 - 0}{40 - 0}$$

$$V_{ave} = \frac{0.3(40)^3}{40} = \frac{0.3(64000)}{40} = \frac{19200}{40} = 480 \text{ ft/s}$$

(C) What is the average velocity of the rocket during the first 1000 ft of its ~~height~~ flight?

First we shall find time taken to reach 1000 ft.

As $s = 0.3t^3$, and $s = 1000$

$$\Rightarrow 0.3t^3 = 1000 \Rightarrow \frac{3}{10}t^3 = 1000$$

$$t^3 = \frac{1000 \times 10}{3}$$

$$t^3 = \frac{10000}{3} \Rightarrow t = \left(\frac{10000}{3}\right)^{1/3}$$

$$t \simeq 14.938$$

For average velocity

$$s = 0.3t^3, \quad t_1 = 0, \quad t_2 = 14.938$$

$$\therefore \quad V_{ave} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$V_{ave} = \frac{s(14.938) - s(0)}{14.938 - 0} \qquad \because s = 0.3(14.938)^3 = 1000$$

$$V_{ave} = \frac{1000 - 0}{14.938} \simeq 66.943 \text{ ft/s}$$

(d) What is the instantaneous velocity of the rocket at the end of 40 s.

$$s(t) = 0.3t^3, \qquad t_0 = 40$$

EXERCISE 2.1

$$V_{inst} = \lim_{t_1 \to t_0} \frac{s(t_1) - s(t_0)}{t_1 - t_0}$$

$$= \lim_{t_1 \to 40} \frac{s(t_1) - s(40)}{t_1 - 40}$$

$$= \lim_{t_1 \to 40} \frac{0.3t_1^3 - 19200}{t_1 - 40}$$

$$s(40) = 0.3(40)^3 = 19200$$

$$= \lim_{t_1 \to 40} \frac{\frac{3}{10} t_1^3 - 19200}{t_1 - 40}$$

$$= \lim_{t_1 \to 40} \frac{\frac{3}{10}(t_1^3 - 64000)}{t_1 - 40}$$

$$= \lim_{t_1 \to 40} \frac{3/10 \, (t_1^3 - 40^3)}{t_1 - 40}$$

$$\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$= \lim_{t_1 \to 40} \frac{3/10 \, (t_1 - 40)(t_1^2 + 40t_1 + 40^2)}{(t_1 - 40)}$$

$$= \lim_{t_1 \to 40} \frac{3/10 \, (t_1 - 40)(t_1^2 + 40t + 1600)}{(t_1 - 40)}$$

$$= \lim_{t_1 \to 40} 3/10 \, (t_1^2 + 40t + 1600)$$

$$= \frac{3}{10}(40^2 + 40(40) + 1600)$$

$$= 3/10 \, (1600 + 1600 + 1600) = \frac{3}{10}(4800) = 3(480)$$

$$= 1440 \text{ ft/s}$$

OR

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EXERCISE 2.1

$$V_{inst} = \lim_{h \to 0} \frac{s(t_0+h) - s(t_0)}{h}$$

$$= \lim_{h \to 0} \frac{0.3(t_0+h)^3 - s(40)}{h}$$

$$= \lim_{h \to 0} \frac{0.3(40+h)^3 - 19200}{h} \qquad s(40) = 19200$$

$$= \lim_{h \to 0} \frac{0.3\left(40^3 + h^3 + 3(40)h(40+h)\right) - 19200}{h}$$

$$= \lim_{h \to 0} \frac{0.3\left(\overset{64000}{\cancel{1600}} + h^3 + 120h(40+h)\right) - 19200}{h}$$

$$= \lim_{h \to 0} \frac{0.3\left(\overset{64000}{\cancel{1600}} + h^3 + 4800h + 120h^2\right) - 19200}{h}$$

$$= \lim_{h \to 0} \frac{0.3\left[64000 + h^3 + 4800h + 120h^2 - 64000\right]}{h}$$

$$= \lim_{h \to 0} \frac{0.3\left[h^3 + 4800h + 120h^2\right]}{h}$$

$$= \lim_{h \to 0} \frac{0.3h\left[h^2 + 4800 + 120h\right]}{h}$$

$$V_{inst} = \lim_{h \to 0} 0.3\left[h^2 + 4800 + 120h\right] = 0.3\left[0 + 4800 + 0\right]$$

$$= 1440 \text{ ft/s}$$

EXERCISE 2.1

28/ An automobile is driven down a straight highway such that after $0 \le t \le 12$ seconds it is $s = 4.5t^2$ feet from its initial position

(a) Find average velocity of the car over the interval $[0, 12]$.

$$S = 4.5t^2, \quad t_1 = 0, \quad t_2 = 12$$

$$V_{ave} = \frac{S(t_2) - S(t_1)}{t_2 - t_1}$$

$$= \frac{S(12) - S(0)}{12 - 0}$$

$$= \frac{4.5(12)^2 - 0}{12} = (4.5)(12) = 54 \text{ ft/s}.$$

(b) Find the instantaneous velocity of the car at $t = 6s$

$$V_{inst} = \lim_{t_1 \to t_0} \frac{S(t_1) - S(t_0)}{t_1 - t_0}$$

$$S(t) = 4.5t^2, \quad t_0 = 6$$

$$S(t_1) = 4.5t_1^2, \quad S(6) = 4.5(6)^2 = 162$$

$$V_{inst} = \lim_{t_1 \to 6} \frac{4.5(t_1)^2 - 4.5(6)^2}{t_1 - 6}$$

$$= \lim_{t_1 \to 6} \frac{4.5t_1^2 - 4.5(6)^2}{t_1 - 6}$$

EXERCISE 2.1

$$V_{inst} = \lim_{t_1 \to 6} \frac{4.5(t_1^2 - 36)}{t_1 - 6}$$

$$= \lim_{t_1 \to 6} \frac{4.5(t_1 + 6)(t_1 - 6)}{(t_1 - 6)}$$

$$= \lim_{t_1 \to 6} 4.5(t_1 + 6) = 4.5(6 + 6) = 54 \text{ ft/s}$$

OR

$$V_{inst} = \lim_{h \to 0} \frac{s(t_0 + h) - s(t_0)}{h}$$

$$= \lim_{h \to 0} \frac{s(6 + h) - s(6)}{h}$$

$$= \lim_{h \to 0} \frac{4.5(6 + h)^2 - 4.5(6)^2}{h}$$

$$= \lim_{h \to 0} \frac{4.5(36 + h^2 + 12h) - 4.5(36)}{h}$$

$$= \lim_{h \to 0} \frac{4.5(36 + h^2 + 12h - 36)}{h}$$

$$= \lim_{h \to 0} \frac{4.5(h^2 + 12h)}{h}$$

$$= \lim_{h \to 0} \frac{4.5h(h + 12)}{h}$$

$$= \lim_{h \to 0} 4.5(h + 12) = 4.5(0 + 12) = 54 \text{ ft/s}$$

29/ A robot moves in the positive direction along a straight line so that after t minutes its distance is $s = 6t^4$ feet from the origin

(a) Find the average velocity of the robot over the interval [2, 4]

$$s = 6t^4, \quad t_1 = 2, \quad t_2 = 4$$

$$s(t_1) = s(2) = 6(2)^4 = 96$$

$$s(t_2) = s(4) = 6(4)^4 = 1536$$

$$V_{ave} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

$$= \frac{s(4) - s(2)}{t_2 - t_1}$$

$$= \frac{6(4)^4 - 6(2)^4}{4 - 2} = \frac{1536 - 96}{2} = \frac{1440}{2} = 720 \text{ ft/min}$$

(b) Find the instantaneous velocity t = 2

$$s(t) = 6t^4; \quad t_0 = 2$$

$$V_{inst} = \lim_{t_1 \to t_0} \frac{s(t_1) - s(t_0)}{t_1 - t_0}$$

$$= \lim_{t_1 \to 2} \frac{s(t_1) - s(2)}{t_1 - 2}$$

EXERCISE 2.1

$$V_{inst} = \lim_{t_1 \to 2} \frac{6t_1^4 - 6(2)^4}{t_1 - 2}$$

$$= \lim_{t_1 \to 2} \frac{6(t_1^4 - 16)}{t_1 - 2}$$

$$= \lim_{t_1 \to 2} \frac{6[(t_1^2)^2 - 4^2]}{t_1 - 2}$$

$$= \lim_{t_1 \to 2} \frac{6(t_1^2 + 4)(t_1^2 - 4)}{t_1 - 2}$$ $\because a^2 - b^2 = (a+b)(a-b)$

$$= \lim_{t_1 \to 2} \frac{6(t_1^2+4)(t_1+2)(t_1-2)}{t_1 - 2}$$ $\because \cancel{t_1^2 - 4 = (t_1-2)(t_1+2)}$

$\because t_1^2 - 4 = (t_1-2)(t_1+2)$

$$= \lim_{t_1 \to 2} 6(t_1^2+4)(t_1+2)$$

$$= 6(2^2+4)(2+2)$$

$$= 6(4+4)(4) = 6(8)(4) = 192 \text{ ft/min}$$